

CP Violation in Heavy Neutrino Mediated $e^-e^- \rightarrow W^-W^{-*}$

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Abstract

We consider the reaction $e^-e^- \rightarrow W^-W^-$ mediated by possible heavy neutrino exchange at future LINAC energies of $\sqrt{s} \gg 2m_W$. This reaction is sensitive to CP phases of the neutrino mixing matrices, even at the level of Born amplitudes. Certain integrated cross-sections are shown to have the power to resolve the CP phases when the experimental configurations are varied. Asymmetries sensitive to CP violation (involving initial QED phases) for e^-e^- and e^+e^+ reactions are constructed and their consequences considered.

Keywords: Neutrino mixing, CP violation, e^-e^- collisions.

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The recent past has witnessed some interest in the construction of novel LINACs that would collide like sign leptons, viz., e^-e^- , e^+e^+ , $\mu^-\mu^-$ and $\mu^+\mu^+$ in addition to conventional e^+e^- collisions, planned into the next century at typical center of mass energies of 500 GeV or even 1 TeV [1]. In principle, the novelty of e^-e^- colliders would include searching for non-resonant phenomena which would be swamped out at e^+e^- colliders and for lepton-number violating phenomena since the net lepton number of the initial state would be 2. In particular, heavy neutrino effects might mediate reactions of the type:

$$e_L^- e_L^- \rightarrow W^- W^-. \quad (1)$$

In particular, a ‘minimal’ framework within which the standard model is extended by an arbitrary set of heavy neutrino flavors (labelled by Λ) with renormalizable couplings (with 3 light flavors) is the ‘minimal mass generating case’ (MMC) [2] wherein reaction (1), e.g. Fig. 1, is dominated by the scattering amplitude for the production of longitudinal W ’s. This reaction at large \sqrt{s} would be directly related via crossing to phenomena such as neutrinoless double beta decay when quark currents are coupled to the W ’s [3]. Furthermore, CP violating effects can creep in through the neutrino mass matrix elements which are in general complex, where not all the phases in the matrix elements are physically unobservable. Here we propose that CP violating effects be looked for in reaction (1) arising from this source. It has been shown that the Born amplitude of Fig. 1 is sufficiently rich so as to exhibit interesting effects when certain integrated cross-sections are considered as experimental configurations are varied [4]. Furthermore, the presence of initial state interaction furnishes a source for the construction of asymmetries between observables of reaction (1) and those of its CP conjugate

$$e^+ e^+ \rightarrow W^+ W^+. \quad (2)$$

We begin with a description of the kinematics of reaction (1)

$$e^-(p_1)e^-(p_2) \rightarrow W^-(p_3)W^-(p_4) \quad (3)$$

with

$$s = (p_1 + p_2)^2, \quad t = (p_1 - p_3)^2, \quad u = (p_1 - p_4)^2 \quad (4)$$

and with $\sqrt{s} \gg m_W$, we have the relation for the cosine of the c. m. scattering angle:

$$\cos \theta \simeq 1 + \frac{2t}{s} \simeq -1 + \frac{2u}{s} \quad (5)$$

which may be conveniently rewritten as

$$\frac{t}{s} \simeq -\sin^2 \theta (\equiv -\zeta), \quad \frac{u}{s} \simeq -\cos^2 \theta (\equiv \zeta - 1) \quad (6)$$

It has been shown that the amplitude T for the reaction (1) maybe written as [2, 5, 6]

$$T = \frac{\sqrt{s}}{v_{ch}^2} \sum_{\Lambda} \left(\frac{t}{t - M_{\Lambda}^2} + \frac{u}{u - M_{\Lambda}^2} \right) \quad (7)$$

where

$$v_{ch} = \left(\frac{1}{2\sqrt{2}G_F} \right)^{1/2} = 174.1 \text{ GeV} \quad (8)$$

and G_F is the Fermi constant.

The differential cross-section with respect ζ defined in eq. (6) is then

$$\frac{d\sigma}{d\zeta} = \frac{1}{32\pi s} \frac{q_f}{q_i} |T|^2 \quad (9)$$

and rewriting (7) in terms of (6) and inserting into (9) we obtain

$$\frac{d\sigma}{d\zeta} = \frac{1}{8\pi v_{ch}^4} \left| \sum_{\Lambda} \xi_{\Lambda} f(\zeta, x_{\Lambda}) \right|^2 \quad (10)$$

where $x_{\Lambda} = M_{\Lambda}^2/s$, $\xi_{\Lambda} = U_{e\Lambda} M_{\Lambda}$ with

$$f(\zeta, x) = \frac{1}{2} \left(\frac{\zeta}{\zeta + x} + \frac{1 - \zeta}{1 - \zeta + x} \right) \quad (11)$$

$$\sum_{\Lambda} \xi_{\Lambda} = 0, \quad (12)$$

The condition above, eq. (12) is a result of the minimal mass generating case [7] which is the framework within which the present effects are discussed. In the conventional, see-saw, however, this condition would not be necessary, but the mixing parameters $U_{e\Lambda}$ would be small. Note that in general the ξ_{Λ} are complex and the interesting effects of possible CP violation reside here.

It is also necessary to introduce a partial wave expansion for $f(\zeta, x)$:

$$f(\zeta, x) = \sum_l (2l+1) a_l(x) P_l(1-2\zeta) \quad (13)$$

where

$$a_l(x) = \delta_{l0} - 2x Q_l(2x+1) \quad (14)$$

where we have used the standard definition for the Legendre polynomial of the second kind $Q_l(z)$:

$$Q_l(z) = \frac{1}{2} \int_{-1}^1 dy \frac{P_l(y)}{z-y} \quad (15)$$

In what follows, we will recall some of the steps already discussed in the literature [4] and present more complete details and discussion. We consider a forward and backward conical section of opening angle θ_0 characterized by Δ defined as:

$$\Delta = \cos \theta_0 \quad (16)$$

We consider the total cross-section in the complement of the section above:

$$\sigma(\Delta) = \int_{\frac{1-\Delta}{2}}^{\frac{1+\Delta}{2}} d\zeta \frac{d\sigma}{d\zeta} \quad (17)$$

We therefore have the result:

$$\sigma(\Delta) = \frac{1}{8\pi v_{ch}^4} \left[|\xi_{\Lambda_1}|^2 I(\Delta, x, x) + \text{Re} (\xi_{\Lambda_1} \xi_{\Lambda_2}^* + \xi_{\Lambda_2} \xi_{\Lambda_1}^*) I(\Delta, x, y) + |\xi_{\Lambda_2}|^2 I(\Delta, y, y) \right] \quad (18)$$

where

$$I(\Delta, x, y) \equiv \int_{\frac{1-\Delta}{2}}^{\frac{1+\Delta}{2}} d\zeta f(\zeta, x) f(\zeta, y) \quad (19)$$

and may be expressed in closed form.

Note that in eq. (18) the 2nd term in the square bracket involves the phases that are now physically observable.

In order to devise a strategy to explore the individual terms contributing to $\sigma(\Delta)$, it would be necessary to perform measurements at (a) varying Δ , and/or at (b) varying \sqrt{s} . We explore the scenarios where \sqrt{s} lies between the masses M_1 and M_2 and compute $I(\Delta, x, x)$, $I(\Delta, x, y)$ and $I(\Delta, y, y)$ for a variety of such scenarios and a variety of Δ . These results are presented pictorially: In Fig. 2, we assumed $M_1^2 : M_2^2 = x : y = 1 : 4$, and present profiles of the I 's as functions of $1/x$, with $1 \leq 1/x \leq 4$. A results of a similar exercises for a ratio $x : y = 1 : 10$ are presented in Fig. 3 and finally for a ratio $x : y = 1 : 90$ in Fig. 4. The function $I(\Delta, x, x)$ is bounded from above by Δ .

We finally turn to the issue of detecting CP violation when an experiment is first performed with e^- beams and then with e^+ beams. Certain integrated cross sections, where the differential cross-section is weighted with Legendre polynomials $P_L(1 - 2\zeta)$ provide candidates from which asymmetries may be constructed. In particular, inserting the partial wave expansion introduced earlier into the differential cross-section and weight the later with the Legendre Polynomial above yields the result:

$$\begin{aligned} \sigma_L \equiv \frac{1}{4} \int_0^1 d\zeta \frac{d\sigma}{d\zeta} P_L(1 - 2\zeta) = \\ \sum_{l, l'} \left(\begin{array}{ccc} l & l' & L \\ 0 & 0 & 0 \end{array} \right)^2 (2l+1)(2l'+1) t_l t_{l'}^* \end{aligned} \quad (20)$$

where we have used the identity for the square of the standard Wigner 3 j symbol:

$$\frac{1}{2} \int_{-1}^1 dy P_l(y) P_m(y) P_n(y) = \left(\begin{array}{ccc} l & m & n \\ 0 & 0 & 0 \end{array} \right)^2 \quad (21)$$

and

$$t_l = \sum_{\Lambda} \xi_{\Lambda} a_l(x_{\Lambda}) = |t_l| e^{i\phi_l} \quad (22)$$

Note that we may define an analogous cross-section when the experiment is performed with right handed positron beams $\overline{\sigma}_L$.

Up until now we have not utilized the presence of the initial state phase due to the Coulomb interaction between the initial state e^- (and the e^+ when the experiment is performed with positron beams) which results in the phase shift:

$$e^{2i\delta_l^C} = \frac{\Gamma(l+1-i\alpha)}{\Gamma(l+1+i\alpha)} \quad (23)$$

For small α , this expression has the Taylor expansion:

$$1 - 2i\alpha\psi(l+1), \quad (24)$$

where $\psi(z)$ is the di-gamma function. Furthermore, the Wigner 3 j symbol implies the triangular rule among the l , m , n and vanishes unless this is satisfied. This implies a selection among the l and l' for given L . Thus, upon including the initial state Coulomb phase, we may consider the following difference:

$$\begin{aligned} \sigma_L^C - \overline{\sigma}_L^C = \\ \sum_{l'} \left(\begin{array}{ccc} l & l' & L \\ 0 & 0 & 0 \end{array} \right)^2 (2l+1)(2l'+1) |t_l t_{l'}| \left[-2 \sin(\phi_l - \phi_{l'}) \sin(\delta_l^C - \delta_{l'}^C) \right] \end{aligned} \quad (25)$$

In particular, for $L = 2$ and assuming the dominance of the two lowest (S- and D-) waves, we have the result:

$$\sigma_2^C - \overline{\sigma}_2^C \simeq -6\alpha |t_2 t_0| \sin(\phi_0 - \phi_2) \quad (26)$$

The resulting asymmetry retaining S- and D- waves is:

$$\frac{\sigma_2^C - \overline{\sigma}_2^C}{\frac{1}{2}(\sigma_2^C + \overline{\sigma}_2^C)} = \frac{-3\alpha \sin(\phi_2 - \phi_0)}{\cos(\phi_2 - \phi_0) + |\frac{5t_2}{7t_0}|}. \quad (27)$$

For a natural range of parameters, such as $\sin(\phi_2 - \phi_0) \sim 0.2 - 0.3$, this asymmetry can be at the level of a percent. This constitutes perhaps the most important result of this work.

We also note here that it might be interesting to look for such effects when production of photons in the initial state occurs. Such a proposal has already been made in the context of quark-quark interactions [8]. Also of interest would be to consider the possibility of final state hard bremsstrahlung. A final interesting possibility which is not ruled out is one where the final state W 's are strongly interacting. Such a strongly interacting final state could lead to the enhancement of the CP violating effects due to large final state phases.

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Figure Captions

Fig. 1. Graphs for the amplitude T. Here \mathcal{N}_Λ labels the heavy neutrino flavor of mass M_Λ .

Fig. 2. The integrals $I(\Delta, x, x), I(\Delta, x, y), I(\Delta, y, y)$ with $x : y = 1 : 4$ as a function of $1/x$, $1 \leq 1/x \leq 4$ for $\Delta = 1.00, 0.75$.

Fig. 3. As in Fig. 2 with $x : y = 1 : 10$ and $1 \leq 1/x \leq 10$.

Fig. 4. As in Fig. 2 with $x : y = 1 : 90$ and $1 \leq 1/x \leq 90$.

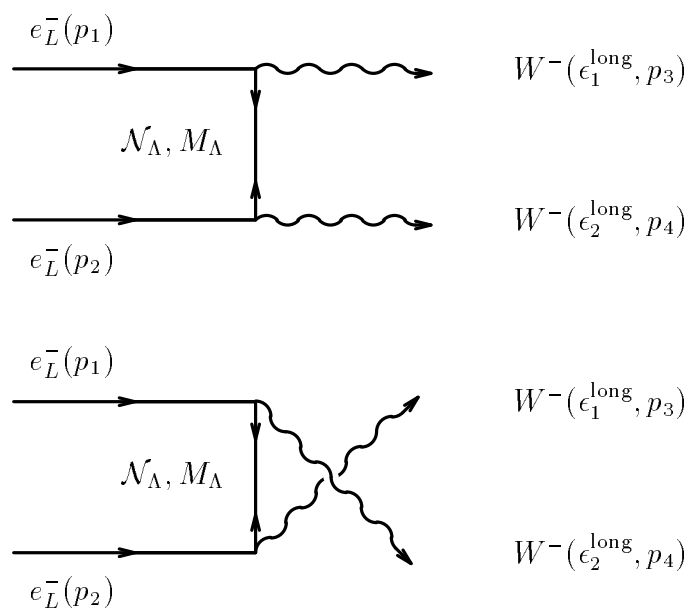


Fig. 1

Fig. 2

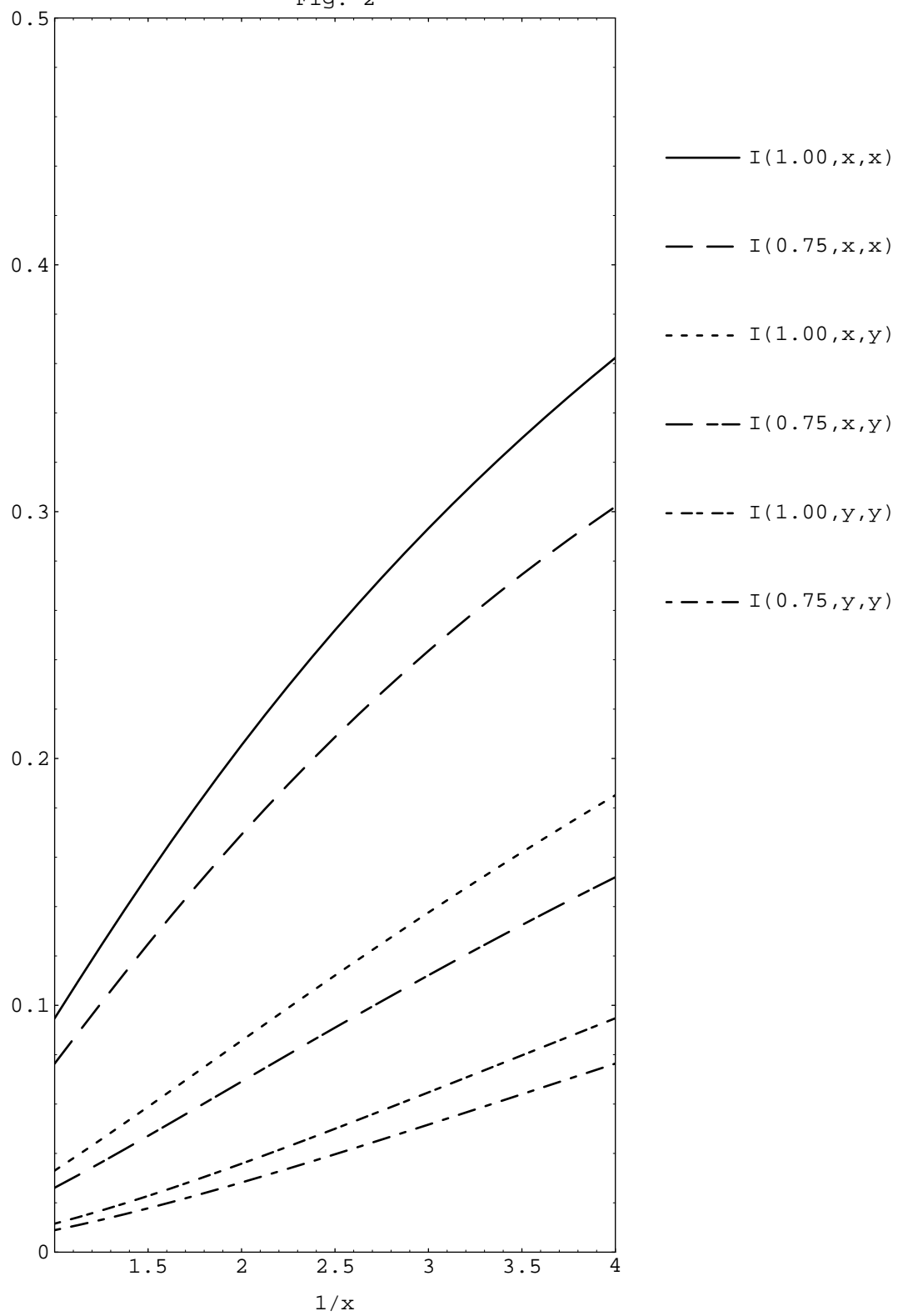


Fig. 3

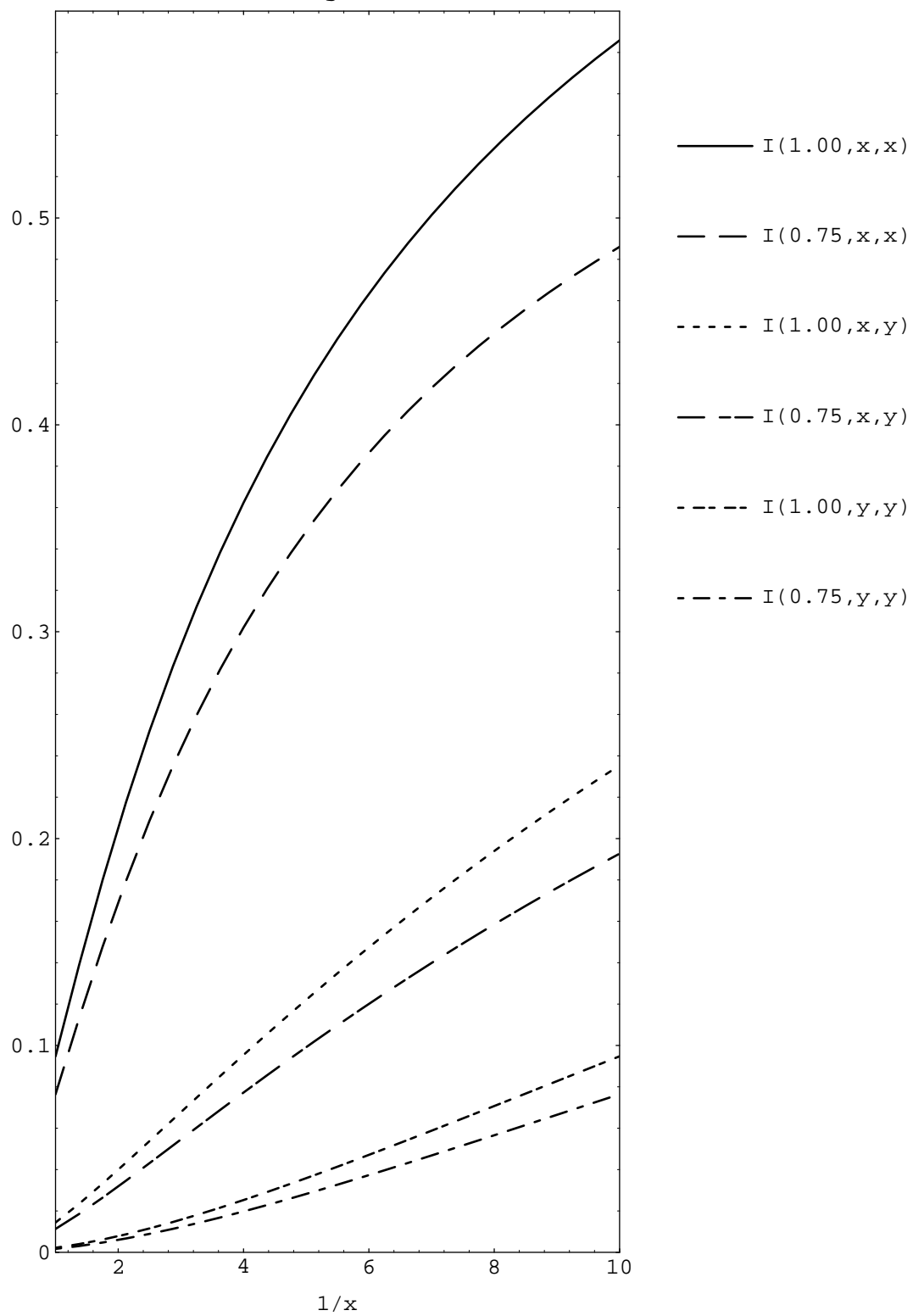


Fig. 4

